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ANALYTICAL PROBABILISTIC MODELING OF URBAN DRAINAGE SYSTEMS

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SUMMARY

The adverse environmental impacts of uncontrolled urban drainage systems are well documented. Informed environmental decision making requires knowledge of impacts resulting from combined sewer overflows (CSOs) and stormwater discharges, the ability of control technologies to mitigate these impacts, and their associated costs. Consequently, the modelling of urban drainage systems is of importance to this process.

While several modeling approaches have been developed to address these issues, each of which have their advantages and disadvantages, they typically require extensive input and significant computational effort. The models presented herein are highly computationally efficient and are intended to enhance planning-level analyses and the development of alternative solutions to urban drainage problems. This modeling approach employs statistics derived from long-term rainfall records to define probability density functions (PDFs) for rainfall characteristics.

Through relatively simple mathematical representations of the transformation of rainfall to runoff and the generation of pollutant washoff from a catchment, the PDFs of runoff characteristics are derived from the PDFs of rainfall characteristics. Control elements, such as storage facilities, have been incorporated into the model developments and the PDFs of system performance variables can similarly be derived. Examples of system performance statistics of importance in analyzing options for CSO control include the average annual number of overflows and the average annual volume and pollutant mass of overflows as well as the volumes and pollutant masses of extreme events. If treatment is employed, statistics such as the level of runoff and pollution control performance are important and can be easily calculated. The real power offered by this modelling approach is the ability to produce meaningful results quickly and easily. This feature enhances the modelling experience by facilitating sensitivity analyses and, through the incorporation of cost functions, system optimization.

INTRODUCTION

Urban infrastructure, including stormwater drainage systems, demands massive commitments of capital for construction, operation and maintenance. In order to adequately maintain a given level of service, such as flood protection and/or water quality control, while minimizing the resources required to do so, models for the performance analysis of stormwater drainage systems are essential. Models are, however, only a representation of the physical reality and simplifying assumptions are invariably required to develop tractable models of drainage system performance. Generally, the greater the degree of accuracy of the model, the greater is its level of complexity and, hence, computational burden. A balance, therefore, must be struck between the accuracy and the simplicity of the model; this balance is dependent on the purpose to be served by the model. That is, the selection of models and modelling complexity should reflect the type of analysis required. The analytical probabilistic models proposed herein are intended for screening and planning level analyses to provide immediate insight into the magnitude of stormwater problems, to illustrate runoff control option trade-offs, and generally to assess system performance.

Event-based design storm methods, especially those using intensity-duration-frequency relations and synthetically generated rainfall patterns, have been traditionally employed in the design and analysis of stormwater conveyance systems (see Figure 1). Such methods, however, cannot reveal information regarding the average, long-term performance of drainage systems, especially those systems employing storage. This understanding can only be gleaned from a continuous analysis of system performance. Furthermore, runoff quality and erosion control performance can only be adequately assessed by continuous analysis as these impacts are largely influenced by smaller, more frequent rainfall events and, hence, are more strongly felt in the long term.

The most commonly employed approach for long term drainage system performance analysis is continuous simulation modeling which transforms long term hyetographs, derived from records of observed rainfall data, into runoff hydrographs and routes these hydrographs through conveyance systems, storage facilities and other control devices. The result is a time series of drainage system responses, including flow rates and storage level fluctuations, which are then statistically analyzed to provide information about the performance of the drainage system (see Figure 1). Continuous simulation models can become very complex and may require relatively large commitments of resources.

As an alternative to screening and planning level analysis of drainage systems by continuous simulation, analytical probabilistic models have been proposed and implemented. These models are developed by preprocessing meteorological records to create probability density functions (PDFs) of meteorological characteristics (e.g. rainfall event volumes, durations, interevent times, etc.) and then transforming these PDFs of system inputs to PDFs of system outputs through the hydrological/hydraulic transformation functions of the catchment and its control devices. Since the PDFs of meteorological inputs are derived from the statistical analysis of long term rainfall records, the mathematically derived PDFs of system outputs reflect the long term performance of the drainage system under analysis.

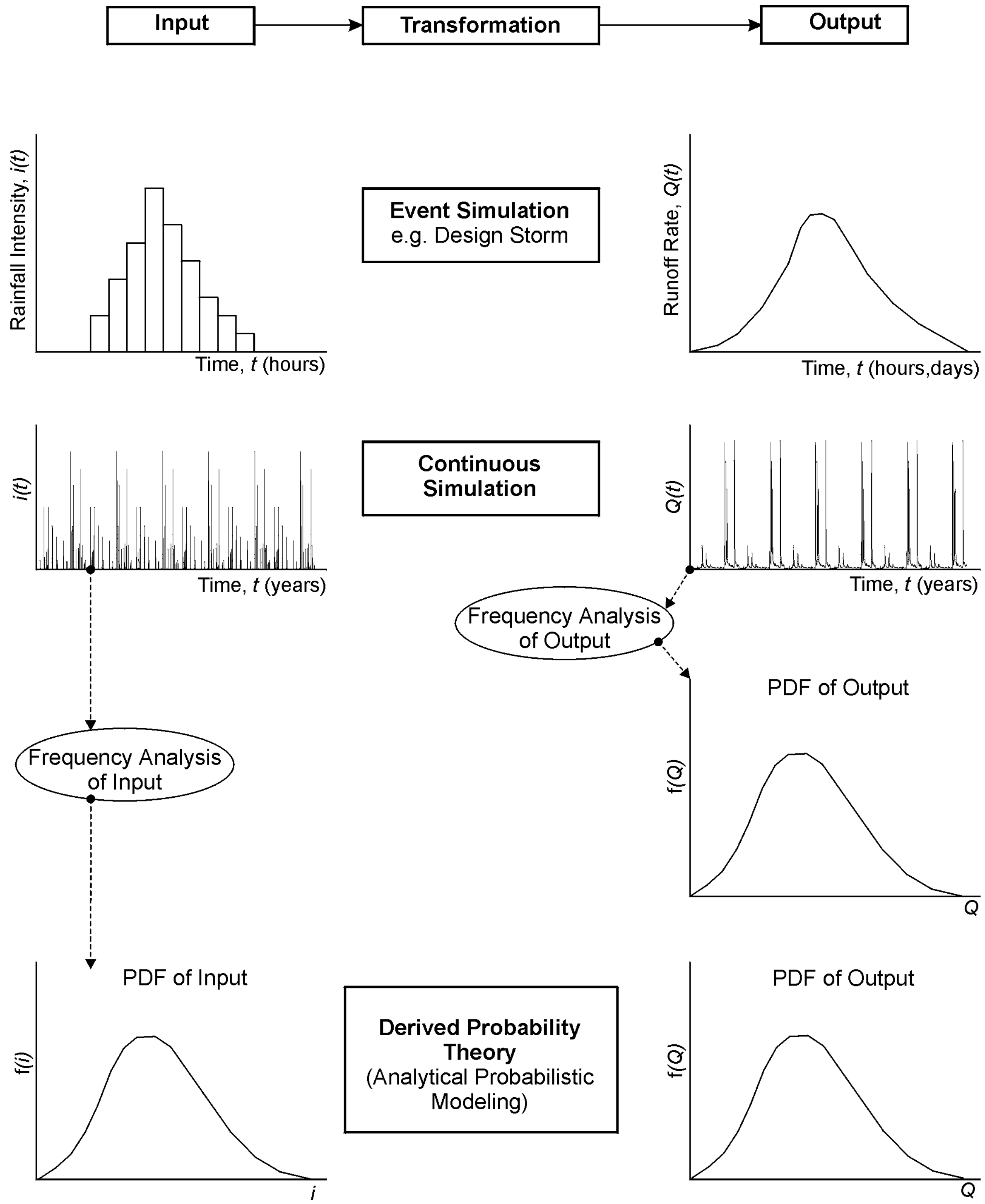

Figure 1. Approaches to Drainage System Analysis

A significant outcome of this process is the derivation of these PDFs, and their associated statistics of runoff control system performance, in mathematically closed form. These algebraic equations explicitly relate the probabilities of performance variables to meteorologic, hydrologic and design parameters thus rendering the exploration of the complete domains of parameter values and design alternatives computationally simple. In order to obtain this mathematical flexibility, analytical models utilize more simplifying assumptions than comparable simulation models. Therefore, when the accuracy of an analysis can be relaxed, permitting the use of relatively simple and, hence, inexpensive analysis techniques, these analytical probabilistic models provide a powerful and comprehensive tool not otherwise available at the screening and planning levels.

PROBLEM TYPES AND MITIGATION MEASURES

As a result of the urbanization of land, runoff volumes and flow rates generally increase and, if left uncontrolled, are liable to cause flooding and erosion problems downstream. Stormwater runoff storage facilities, such as detention ponds and underground tanks, are an effective means of attenuating peak runoff rates such that they are more compatible with the capacity of the downstrean drainage system. Erosion impacts can be mitigated by altering the distribution of flow durations through the use of storage facilities and outlet flow controls.

Sewers susceptible to stormwater flows are liable to experience surcharge conditions and, where such sewers have residential house connections, may thus result in basement flooding. In the case of combined sewer systems, sewer separation and the introduction of storage reservoirs are common mitigative measures.

Water quality degradation resulting from urban drainage, in the form of combined sewer overflows and storm sewer discharges, is a topic which has received much recent attention. Combined sewer overflow volumes may be reduced by employing storage reservoirs which release flows at rates more compatible with the combined sewer system and are ultimately treated at wastewater treatment plants. The detention provided in storage facilities can be further capitalized upon by providing treatment to stormwater runoff and/or combined sewage. Typical removal mechanisms primarily include sedimentation, bacterial decay and biological uptake.

ANALYSIS NEEDS

There is a wide range of alternatives to mitigate the problems associated with urban drainage systems. Different alternatives are more or less effective at mitigating different drainage system problems to different degrees. The challenge to the engineer/analyst is to make decisions on which alternative (or combination of alternatives) to deploy, at what scale of implementation, and under what circumstances. This decision requires information of the effectiveness of the alternatives, and their costs, where effectiveness is usually determined by performance modelling of the alternatives. Performance models must be capable of predicting behaviour under extreme events as well as for annual average conditions; hence, the need for continuous analysis of urban drainage systems.

CATCHMENT MODEL

The hydrologic models developed herein are intended for screening-level analysis. It is important to understand the simplifying assumptions on which they are based. The first assumption regards the nature of the drainage system and the mechanics of water movement, which are illustrated in Figure 2.

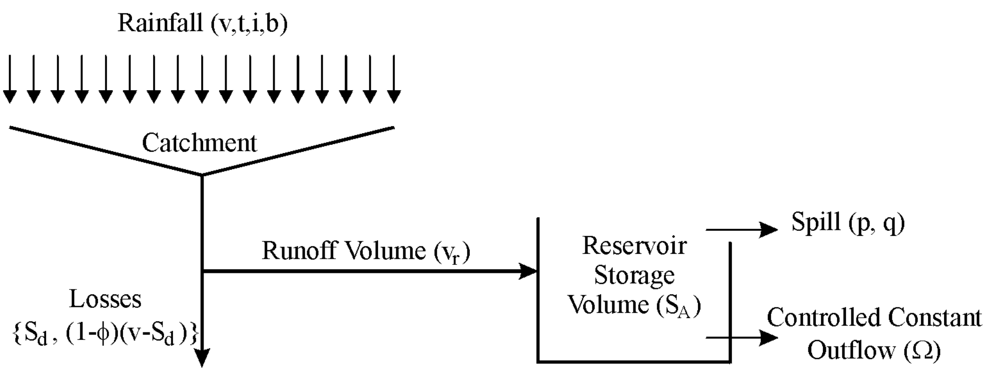

Figure 2. Schematic model of urban drainage systems.

Beginning with the meteorological input to the catchment, it is assumed to be represented by the exponential PDFs of the rainfall characteristics. The catchment then transforms the rainfall volume, v (mm), to runoff volume, v_r (mm), according to the following relationship, which is the hydrologic presentation employed by the STORM simulation model (U.S. Army Corps of Engineers, 1974) – hence, the models derived from the relationship are referred to as the ASTORM models (for Analytical STORM):

$$v_r = \begin{cases} 0 & ; v \le S_d \\ f(v - S_d) & ; v > S_d \end{cases} \tag{1}$$

where rainfall must fill the volume of depression storage, S_d (mm), before runoff occurs. For rainfall volumes above S_d, the runoff volume is given by a product of a dimensionless runoff coefficient, f, and the excess of rainfall over depression storage, (v – S_d). The volume (1 - f)(v - S_d) may be viewed as a uniform infiltration loss occurring after the initial depression storage loss and lasting until the end of the event. The runoff coefficient, f, is a spatially and temporally averaged constant which is selected on the basis of land use, soil type, and topography. It is assumed that the duration of the runoff event, t_r, is practically equal to the duration of the rainfall event, t. It is also noted that the full depression storage, S_d, is assumed to be available at the beginning of every rainfall event since the volume of depression storage on urban catchments is typically small.

Once the runoff is generated as a square-wave hydrograph, it is translated to the reservoir instantaneously, without change in shape. The rationale for this assumption is that for runoff control systems incorporating storage, the system performance is influenced more by the runoff volume and duration than by the exact shape of the runoff hydrograph. It is noted that the models presented herein are intended for the analysis of urban drainage systems with storage. The design of drainage conveyances (e.g., pipes, channels), however, depends on the peak rate of flow and are thus sensitive to the shape of the runoff hydrograph. For such problems, alternative analytical models of the ASWMM Type (e.g., Guo and Adams, 1998a, 1998b; 1999a, 1999b, 1999c) or alternative simulation models would be more appropriate.

The maximum storage volume of the reservoir, S_A (mm), is expressed as the equivalent depth across the catchment area. The reservoir is drained by a controlled outflow at a constant rate, W (mm/h), also expressed as equivalent depth across the catchment area. When the reservoir is full and the inflow occurs at a greater rate than the outflow, the excess volume, p (mm), is spilled. Such spills may be visualized as occurring upstream of the storage facility (through a flow splitter/regulator) or as being routed through the storage facility.

PROBABILITY DENSITY FUNCTIONS OF RAINFALL EVENT CHARACTERISTICS

Characteristics of rainfall events of interest in drainage systems analysis are: event volume (v), event duration (t), average intensity of event (i), and the interevent time (b) which is the dry period between successive rainfall events. In order to develop the probability density functions (PDFs) of these characteristics, individual rainfall events are identified by the criterion of the minimum interevent time definition (IETD). Events with interevent times greater than or equal to this minimum are classified as distinct events. With this criterion, a long term rainfall record (typically decades in length) is parsed into events. The samples of values of each characteristic are then subjected to a statistical analysis revealing their moments and coefficients. Sample histograms and/or cumulative distribution functions (CDFs) are plotted and theoretical distributions are fitted.

In this work, exponential distributions are fitted to the sample data as summarized in Table 1. In addition to the numerical values of z, l, b and y (the inverses of the mean rainfall event volume, duration, average intensity and interevent time, respectively), the value of q, the average annual number of rainfall events, is also known from the statistical analysis of the rainfall record.

Table 1. PDFs of rainfall characteristics

Rainfall Characteristics	Exponential PDF		Applicable Range
Volume, v (mm)	$f_V(v) = z\, e^{-z v}$	$z = \frac{1}{\bar{v}}$	$0 \le v \le \infty$
Duration, t (h)	$f_T(t) = l\, e^{-l t}$	$l = \frac{1}{\bar{t}}$	$0 \le t \le \infty$
Average intensity, i (mm/h)	$f_I(i) = b\, e^{-b t}$	$b = \frac{1}{\bar{i}}$	$0 \le i \le \infty$
Interevent time, b (h) Simplified version	$f_B(b) = y\, e^{-y(b-IETD)}$ $f_B(b) = y\, e^{-y b}$	$y = \frac{1}{\bar{b}\text{-}IETD}$ $y = \frac{1}{\bar{b}}$	$IETD \le b \le \infty$ $0 \le b \le \infty$

DERIVED PROBABILITY DISTRIBUTION THEORY

To illustrate the principle of derived probability distribution theory, a simple case of a function of a single random variable is discussed below.

Consider a random variable X with a known probability density function (PDF), $f_X(x)$. Also consider a monotonically increasing function, $y = g(x)$, which transforms values of X to values of Y in one-to-one correspondence (i.e., only one value of Y exists for every value of X and vice versa). Then Y is a random variable whose PDF, $f_Y(y)$, can be derived as follows (see Figure 3).

If $y = g(x)$ [therefore, $x = g^{-1}(y)$] is monotonically increasing and maps $X \to Y$ one-to-one, then

$$F_Y(y) = \text{Prob}[Y \le y] = \text{Prob}\left[X \le g^{-1}(y)\right] = F_X\left[g^{-1}(y)\right] \tag{2}$$

where $F_Y(y)$ is the cumulative distribution function (CDF) of Y. By definition, the PDF is the first derivative of the CDF, or

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X\left[g^{-1}(y)\right] = \frac{d}{dy}\int_{-\infty}^{g^{-1}(y)} f_X(x)\,dx \tag{3}$$

Using Leibniz's rule for differentiating an integral, Equation 3 is equivalent to

$$f_Y(y) = \frac{d}{dy} g^{-1}(y) \cdot f_X\left[g^{-1}(y)\right] \tag{4}$$

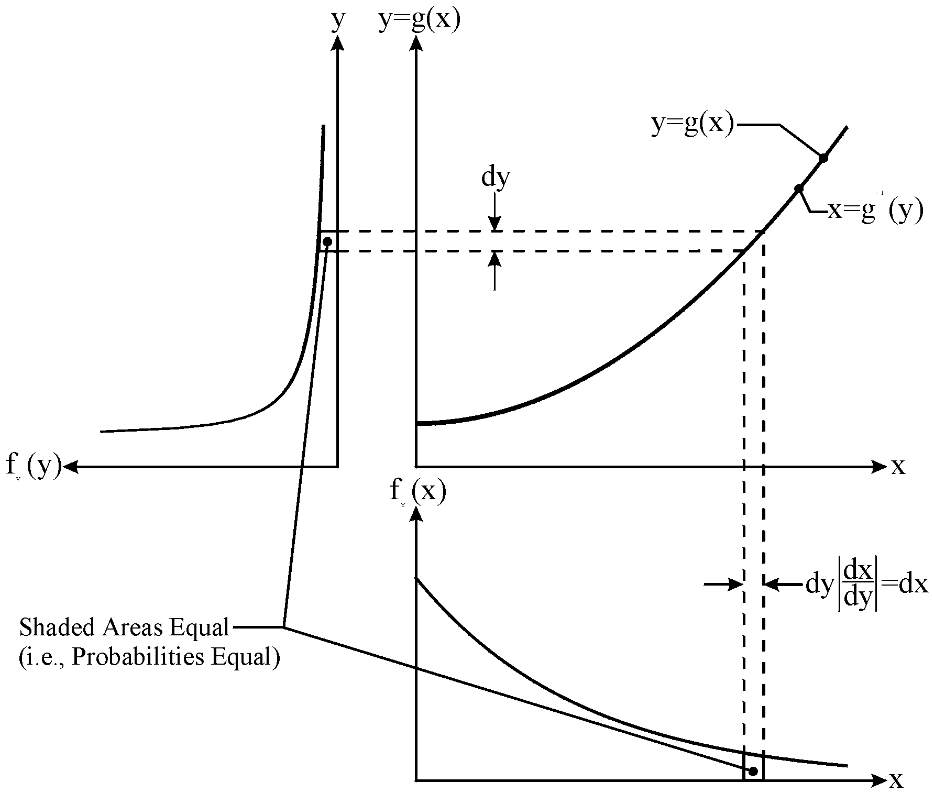

Figure 3. Graphical interpretation of derived probability distribution of a monotonic function of one random variable with a one-to-one mapping. (After Benjamin and Cornell, 1970).

or, since $x = \mathrm{g}^{-1}(y)$

$$\mathrm{f}_Y(y) = \frac{dx}{dy}\mathrm{f}_X(x) \tag{5}$$

Restating Equation 5 yields

$$\mathrm{f}_Y(y)dy = \mathrm{f}_X(x)dx \tag{6}$$

The result above is illustrated graphically in Figure 3. If $y = \mathrm{g}(x)$ maps $X \rightarrow Y$ one-to-one but is monotonically decreasing, then Equations 4 and 5 are modified respectively to

$$\mathrm{f}_Y(y) = \left|\frac{d}{dy}\mathrm{g}^{-1}(y)\right|\mathrm{f}_X\left[\mathrm{g}^{-1}(y)\right] \tag{7}$$

and

$$\mathrm{f}_Y(y) = \left|\frac{dx}{dy}\right|\mathrm{f}_X(x) \tag{8}$$

EXAMPLE OF DERIVED PROBABILITY DISTRIBUTION – RUNOFF VOLUME

Given the marginal PDF of rainfall volume (Table 1), and the runoff model (Equation 1), the cumulative distribution function (CDF) of runoff volume, $F_{V_r}(v_r)$, may be obtained using derived probability distribution theory. From the CDF of runoff volume, the PDF of runoff volume, $f_{V_r}(v_r)$, may be obtained by differentiation. Figure 4 schematically shows the transformation of the PDF of rainfall volume to the PDF of runoff volume according to the rainfall-runoff transformation function as depicted in Equation 1. According to the rainfall-runoff model, some rainfall events will not cause runoff events if their volume is less than the depression storage. As a result, there is an impulse probability that no runoff will occur which is equal to the probability that a given rainfall event's volume does not exceed depression storage which is represented by the shaded area in Figure 4. This impulse probability is given by

$$p_{V_r}(0) = \text{Prob}[V_r = 0] = \text{Prob}[V \le S_d] = \int_{v=0}^{S_d} f_V(v)\,dv = \int_{v=0}^{S_d} z e^{-zv}\,dv = 1 - e^{-zS_d} \qquad (9)$$

The remainder of the CDF of runoff volume exists over the range where runoff occurs ($v_r > 0$) which corresponds to the range where the volume of a rainfall event is greater than the depression storage value ($v > S_d$). That is,

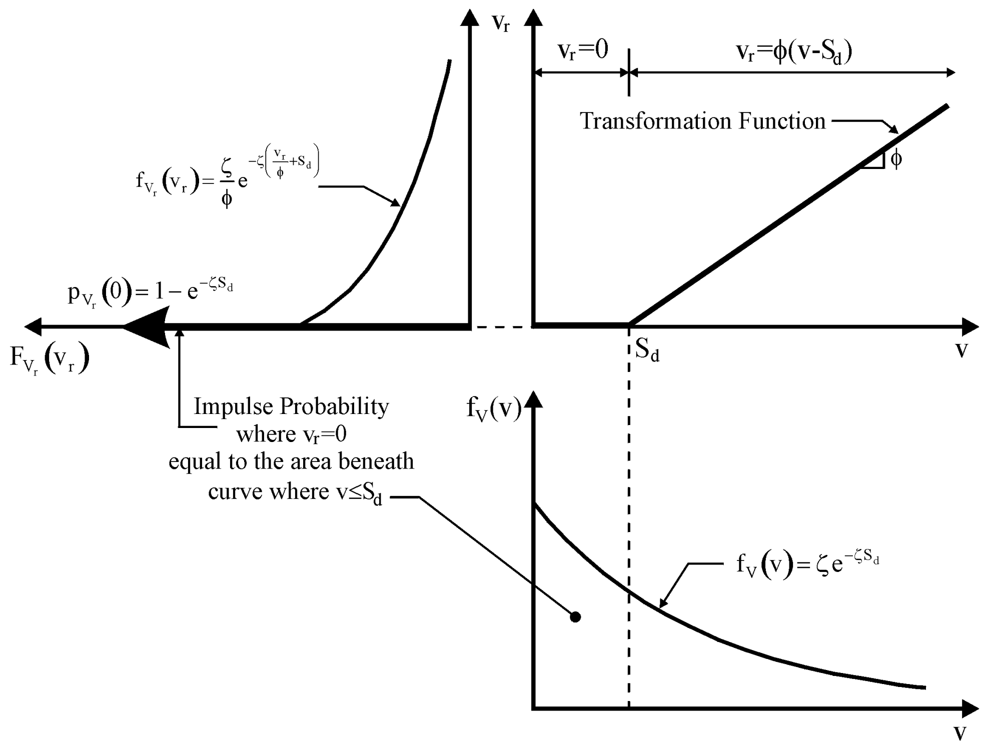

Figure 4. Transformation of PDF of rainfall volume to PDF of runoff volume

$$\begin{aligned}\mathrm{F}_{V_r}(v_r) &= \mathrm{Prob}[V_r \le v_r] = \mathrm{Prob}(V_r = 0) + \mathrm{Prob}\left[S_d < V \le \frac{v_r}{f} + S_d\right] \\ &= \mathrm{Prob}(V_r = 0) + \int_{S_d}^{\frac{v_r}{f}+S_d} \mathrm{f}_V(v)dv \\ &= \mathrm{Prob}(V_r = 0) + \left(1 - \mathrm{e}^{-\frac{z}{f}v_r}\right)\mathrm{e}^{-zS_d} = 1 - \mathrm{e}^{-z\left(\frac{v_r}{f}+S_d\right)}\end{aligned} \quad (10)$$

over the range where $v_r > 0$.

The PDF of runoff volume may be obtained as the derivative of Equation 10 as follows:

$$\mathrm{f}_{V_r}(v_r) = \frac{d}{dv_r}\mathrm{F}_{V_r}(v_r) = \frac{d}{dv_r}\left[1 - \mathrm{e}^{-z\left(\frac{v_r}{f}+S_d\right)}\right] = \frac{z}{f}\mathrm{e}^{-z\left(\frac{v_r}{f}+S_d\right)} \quad ;\, v_r > 0 \quad (11)$$

The impulse probability associated with $v_r = 0$ is noted and is given by Equation 9.

SYSTEM PERFORMANCE MODELS

As with the derivation of the PDF of runoff volume, derived probability distribution theory is used to develop the PDFs of numerous performance variables that describe the behaviour of the catchment and the runoff quantity and quality control system. In addition to the full PDFs of the performance variables, the event means and the average annual means of the performance variables are also derived.

A summary of the resultant performance models is given below. The mathematical derivations of these models is given by Adams and Papa (2000). It is noted that the performance models are derived for two sets of assumptions regarding reservoir contents: (1) reservoir full at the end of the last event ($s_i = S_A$) and (2) reservoir empty at end of last event ($s_i = 0$). In addition, two sets of equations are derived for the two forms of the PDF for interevent time as given in Table 1. The summary below includes the performance models developed for the simpler PDF of interevent time.

PRECIPITATION

Average annual precipitation volume

$$P_p = \frac{q}{z} \quad (12)$$

RUNOFF

Average annual volume of runoff

$$R = q\,\frac{f}{z}\,\mathrm{e}^{-zS_d} \qquad (13)$$

Average annual number of runoff events

$$n_r = q \cdot \mathrm{e}^{-zS_d} \qquad (14)$$

Average annual loss volume

$$L = \frac{q}{z}\left(1 - f \cdot \mathrm{e}^{-zS_d}\right) \qquad (15)$$

Average annual volume of depression storage loss

$$D_a = \frac{q}{z}\left(1 - \mathrm{e}^{-zS_d}\right) \qquad (16)$$

RUNOFF QUANTITY CONTROL

Probability per rainfall event of spill with a magnitude of at least p_0 (assuming that $s_i = S_A$)

$$G_P(p_0) = \left[\frac{\frac{l}{\Omega}}{\frac{l}{\Omega} + \frac{z}{f}}\right]\left[\frac{\frac{y}{\Omega} + \frac{z}{f}\, e^{-\left(\frac{y}{\Omega} + \frac{z}{f}\right)S_A}}{\frac{y}{\Omega} + \frac{z}{f}}\right]\mathrm{e}^{-z\left(\frac{p_0}{f} + S_d\right)} \qquad (17)$$

Probability per rainfall event of spill with a magnitude of at least p_0 (assuming that $s_i = 0$)

$$G_P(p_0) = \left[\frac{\frac{l}{\Omega}}{\frac{l}{\Omega} + \frac{z}{f}}\right]\mathrm{e}^{-z\left(\frac{p_0}{f} + \frac{S_A}{f} + S_d\right)} \qquad (18)$$

Probability per rainfall event of any spill occurring (assuming that $s_i = S_A$)

$$G_P(0) = \left[\frac{\frac{l}{\Omega}}{\frac{l}{\Omega} + \frac{z}{f}}\right]\left[\frac{\frac{y}{\Omega} + \frac{z}{f}\, \mathrm{e}^{-\left(\frac{y}{\Omega} + \frac{z}{f}\right)S_A}}{\frac{y}{\Omega} + \frac{z}{f}}\right]\mathrm{e}^{-z \cdot S_d} \qquad (19)$$

Probability per rainfall event of any spill occurring (assuming that $s_i = 0$)

$$\mathrm{G}_P(0) = \left[\frac{\frac{l}{\Omega}}{\frac{l}{\Omega} + \frac{z}{f}} \right] \mathrm{e}^{-z\left(\frac{S_A}{f} + S_d\right)} \tag{20}$$

Average annual number of spills

$$n_s = q \cdot \mathrm{G}_P(0) \tag{21}$$

Average annual spill volume

$$P_u = q \frac{f}{z} \mathrm{G}_p(0) = \frac{f}{z} n_s \tag{22}$$

Spill volume of specified return period, T_R (years)

$$p_{T_R} = \frac{f}{z} \ln\left[q T_R \mathrm{G}_P(0)\right] \tag{23}$$

Average annual fraction of runoff lost to spills

$$R_s = \frac{P_u}{R} \tag{24}$$

Average annual fraction of runoff controlled

$$C_R = 1 - \frac{P_u}{R} = 1 - \mathrm{G}_P(0) e^{zS_d} \tag{25}$$

Isoquant of average annual number of spills, n_s

$$S_A = -\frac{\Omega f}{y f + z\Omega} \ln\left\{ \frac{f}{z} \left[\frac{n_s}{q} \left(1 + \frac{z\Omega}{l f}\right) \left(\frac{y}{\Omega} + \frac{z}{f}\right) e^{z S_d} - \frac{y}{\Omega} \right] \right\} \text{ assuming } s_i = S_A \tag{26}$$

$$S_A = -\frac{f}{z} \ln\left[\frac{n_s}{q} \left(1 + \frac{z\Omega}{l f}\right) \right] - f S_d \text{ assuming } s_i = 0 \tag{27}$$

Isoquant of average annual fraction of runoff controlled, C_R

$$S_A = -\frac{\Omega f}{y f + z\Omega} \ln\left\{ \frac{f}{z} \left[(1 - C_R)\left(1 + \frac{z\Omega}{l f}\right)\left(\frac{y}{\Omega} + \frac{z}{f}\right) - \frac{y}{\Omega} \right] \right\} \text{ assuming } s_i = S_A \tag{28}$$

$$S_A = -\frac{f}{z} \ln\left[(1 - C_R)\left(1 + \frac{z\Omega}{l f}\right) \right] \text{ assuming } s_i = 0 \tag{29}$$

Average annual number of runoff events that do not utilize storage

$$n_p = q\, e^{-b\Phi}\left(1 - e^{-\frac{b}{f}\Omega}\right) \text{ where } \Phi = \frac{z}{b} S_d \tag{30}$$

Average annual volume of runoff processed without storage

$$P_w = \frac{q\, f\, e^{-b\Phi}}{l\, b}\left[1 - \left(1 + \frac{b\,\Omega}{f}\right) e^{-\frac{b\,\Omega}{f}}\right] \text{ where } \Phi = \frac{z}{b} S_d \tag{31}$$

or

$$P_w = R - P_u = q\,\frac{f}{z}\left[e^{-z\, S_d} - G_P(0)\right]$$

RUNOFF QUALITY CONTROL

Combined pollution removal efficiency

$$h = h_S + h_\Omega (1 - h_S) = h_S + h_\Omega - h_S h_\Omega \tag{32}$$

Average annual fraction of pollution control

$$C_P = \frac{(R - P_u) h + P_u T h_S}{R} \tag{33}$$

Average steady-state detention time of storage facility

$$t_s = \frac{1}{2} t_d = \frac{1}{2} \frac{S_A}{\Omega} \tag{34}$$

Overall fractional TSS removal efficiency under dynamic settling conditions

$$E_d = \sum_i F_i \left[1 - \left(1 + \frac{V_{S_i}}{n\,h_A} \frac{S_A}{2\Omega} \right)^{-n} \right] \tag{35}$$

Overall fractional TSS removal efficiency under quiescent settling conditions

$$E_q = \sum_i F_i \left[\frac{V_{S_i}}{y\ h_P} \left(1 - e^{-\frac{y\ h_P}{V_{S_i}}} \right) \right] \tag{36}$$

Average annual fraction of pollution control from extended detention dry pond

$$C_P = E_d\, C_R = E_d \left[1 - G_P(0) e^{z\ S_d} \right] \tag{37}$$

Average annual fraction of pollution control from wet pond without outlet control

$$C_P = E_q \left(1 - e^{-\frac{z}{f} S_P} \right) \tag{38}$$

Average annual fraction of pollution control from wet pond with outlet control (i.e. extended detention wet pond)

$$C_P = E_q \left(1 - e^{-\frac{z}{f} S_P} \right) + \frac{(P_u - P'_u) E_d}{R} \tag{39}$$

where

$$P'_u = q\, \frac{f}{z} G_P(0) \tag{40}$$

and $G_P(0)$ is calculated using "effective" depression storage

$$S_{d_{effective}} = S_d + \frac{S_P}{f} \tag{41}$$

The mathematically compact form of the above model expressions for system performance offers exceptional ease in generating results and performing sensitivity analyses in order to gain an overall understanding of system behaviour and illustrating control option trade-offs. Furthermore, these performance models of stormwater management control alternatives may be easily combined with economic functions of system design variables in an optimization context. The result of this exercise is the formulation of methodologies for the cost-effective selection of design alternatives and scale of implementation of alternatives to meet runoff quantity and quality control objectives of urban drainage systems.

VALIDITY OF PERFORMANCE MODELS

The above performance models are developed with a number of simplifying assumptions for reasons of mathematical tractability. The developments generally follow the processes of hydrologic simulation models (i.e., the STORM model in the present work) but can never include the level of complexity allowed by simulation modeling. However, this is consistent with the intended use of the analytical performance models for system planning, screening level analysis, and preliminary design. Nevertheless, it is important to understand the limits of accuracy and the range of applicability of the models. Therefore, extensive comparisons have been undertaken between the results obtained from the analytical performance models and the equivalent results from continuous simulation models (e.g., Seto, 1984; Kauffman, 1987; Papa et al., 1997; Li and Adams, 2000). In general, favourable comparisons have been found for many climatic regions. These results suggest that analytical performance models for runoff quantity and quality control planning offer a powerful alternative/supplement to continuous simulation modeling.

CONCLUSIONS

This paper presents a summary of a suite of analytical probabilistic models currently available for the planning level analysis and design of urban drainage systems. These performance models address the most fundamental problems faced by the engineer/planner. Their mathematical closed form allows for extreme computational efficiency and ease of implementation. They provide immediate insight into the performance of urban drainage systems and the effectiveness of runoff control alternatives in mitigating typically encountered problems such as flooding, erosion and water quality degradation. These attributes enhance the engineering analysis of urban drainage systems by encouraging the comprehensive analysis of wide ranges of system design alternatives which, in turn, improves the cost effectiveness of system design.

These models are applied in a companion paper (Papa and Adams, 2002) to a variety of problem types.

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NOTATION

A	catchment area (ha)
b	interevent time (h)
$\bar{b}$	average interevent time (h)
C_P	average annual fraction of pollution controlled
C_R	average annual fraction of runoff controlled
D_a	average annual volume of depression storage loss (mm)
E_d	overall TSS removal efficiency in active storage zone under dynamic settling
E_q	overall TSS removal efficiency in permanent pool under quiescent settling
F_i	fraction of total mass contained in i^{th} size fraction
$F_X(x)$	CDF of the continuous random variable X

$f_X(x)$	PDF of the continuous random variable X
$G_P(0)$	probability per rainfall event that any magnitude of spill from storage will occur
$G_P(p_o)$	probability per rainfall event that the volume of spill equals or exceeds p_o
$G_X(x)$	complementary CDF of the continuous random variable X
h_A	depth of active storage zone (m)
h_P	depth of permanent pool (m)
i	average intensity of rainfall event (mm/h)
$\bar{i}$	average of average intensity of rainfall events (mm/h)
i_e	runoff intensity (mm/h)
L	average annual loss volume (mm)
n_p	average annual number of runoff events which do not utilize storage
n_r	average annual number of runoff events
n_s	average annual number of spills
p	volume of runoff spilled from reservoir (mm)
P_u	average annual volume of spills (mm)
P_w	average annual volume of runoff processed without storage (mm)
R	average annual volume of runoff (mm)
R_S	average annual fraction of runoff spilled
S_A	maximum (active) storage volume of downstream reservoir (mm)
S_d	depression storage (mm)
s_i	volume of contents in storage at the end of the i^{th} (previous) event (mm)
S_P	storage capacity of a permanent pool or of a storage reservoir with no outlet (mm)
t	duration of rainfall event (h)
$\bar{t}$	average duration of rainfall events (h)
T	fraction of pollution removal efficiency received by spills routed through storage
t_d	drawdown time of reservoir (h)
t_r	duration of runoff event (h)
$\bar{t}_r$	average duration of runoff event (h)
T_R	return period (yr)
t_s	average steady-state detention time of the pond (h)
v	total volume of rainfall event (mm)
$\bar{v}$	average total volume of rainfall events (mm)
v_r	volume of runoff (mm)
V_S	terminal settling velocity of a discrete particle size (m/h)
V_{S_i}	average settling velocity of particles in the i^{th} size fraction (m/h)
b	parameter for exponential PDF of average rainfall intensity (h/mm)
g	parameter for generic exponential PDF
z	parameter for exponential PDF of rainfall volume (mm^{-1})
h	pollution removal efficiency of runoff controlled by storage/treatment system
h_S	pollution removal efficiency of storage facility
h_W	pollution removal efficiency of treatment facility

l	parameter for exponential PDF of rainfall duration (h^{-1})
q	average annual number of rainfall events
F	rate of loss of depression storage averaged across storm duration (mm/h)
f	runoff coefficient
y	parameter for exponential PDF of interevent time (h^{-1})
W	controlled constant outflow rate of reservoir (mm/h)