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APPLICATION OF ANALYTICAL PROBABILISTIC MODELS FOR URBAN DRAINAGE SYSTEMS ANALYSIS

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SUMMARY

This paper presents illustrative examples of the use of analytical probabilistic modeling of urban drainage systems, the purpose of which is to show the reader how this modeling technology can be employed to provide insight into the magnitude of urban drainage system problems and their solutions. The models are generally efficient, closed-form analytical expressions developed using derived probability distribution theory in conjunction with functions which represent the transformation of rainfall to runoff and the hydraulic processing of runoff through drainage conveyance and storage elements. Input to these models includes the basic hydrologic parameters of runoff coefficient and depression storage, as well as rainfall event statistics, derived from the analysis of long-term rainfall records. The former of these input parameters are typically easily derived by the analyst while the latter have been developed for many locations in North America. Continued interest in this modeling approach will see rainfall event statistics derived for more locations around the world.

These models have been developed in response to the need for a simpler, desktop method of calculating the long-term performance of urban drainage systems such as combined sewer and/or storm sewer networks. At present, the understanding of long-term performance is typically derived from continuous simulation over long periods of rainfall data. While this approach has its merits and place in the analyst's toolbox, it is relatively cumbersome for screening or planning-level analyses.

Rather than discussing the theoretical underpinnings of this modeling approach, which is the topic of a companion paper (Adams and Papa, 2002), this paper takes the reader through two examples of the application of these models in order to develop a level of comfort with the capabilities and the use of this technology. The examples indicate how meaningful results can be obtained with relatively little effort, not only in terms of producing single-valued results, but also in terms of producing ranges of results for the purposes of sensitivity analysis and optimization.

ILLUSTRATIVE EXAMPLES

COMBINED SEWER OVERFLOW (CSO) CONTROL

Consider a combined sewer catchment of 400 hectares in area located in Toronto, Canada and which is represented schematically in Figure 1. At the downstream end of the system is an interceptor sewer that conveys flows equivalent to 2 × DWF (dry weather flow) to a wastewater treatment plant. Flows in excess of this capacity are conveyed directly to Lake Ontario and are referred to as Combined Sewer Overflows (CSOs).

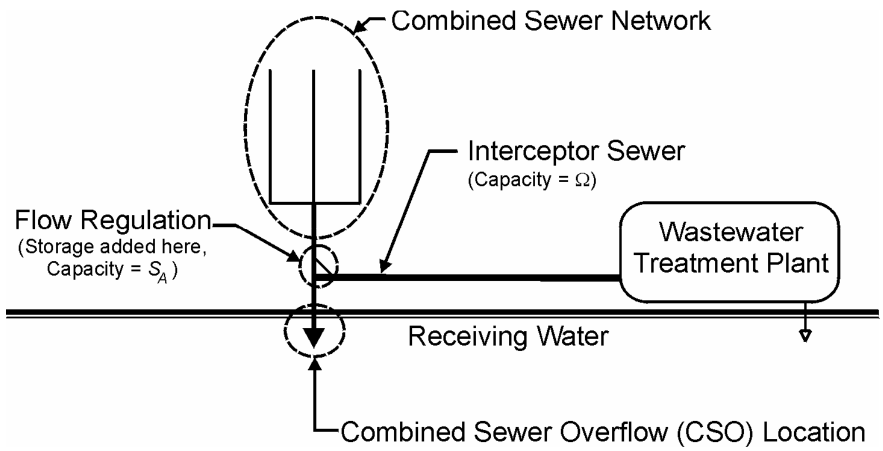

Figure 1. Schematic representation of combined sewer network.

The average population density and domestic wastewater generation rates are assumed to be 100 persons/ha and 400 L/c/d, respectively. The corresponding dry weather flow rate is 16,000 m^3/d or 0.167 mm/h, normalized over the catchment area. Correspondingly, the interceptor sewer capacity is 0.333 mm/h (i.e. 2 × 0.167 mm/h). The residual capacity in the interceptor sewer to convey storm drainage (Ω) is then 1 × DWF (i.e. 2 × DWF – DWF) or 0.167 mm/h.

Flow monitoring studies have been undertaken to understand the drainage system's response to rainfall and from which the following hydrological modeling parameters have been derived:

- runoff coefficient, $\phi = 0.50$
- depression storage, $S_d = 2.0$ mm
- average concentration of total suspended solids (TSS) in CSOs = 200 mg/L

The statistical analysis of numerous years of rainfall events yields the following mean values of several rainfall characteristics and their corresponding exponential probability density function (PDF) parameters (in parentheses):

- mean rainfall event volume, $\bar{v}$ = 5.00 mm (ζ = 0.200 mm^{-1})
- mean rainfall event duration, $\bar{t}$ = 3.55 h (λ = 0.282 h^{-1})
- mean rainfall interevent time, $\bar{b}$ = 43.4 h (ψ = 0.0230 h^{-1})
- average annual number of rainfall events, θ = 104

Several meaningful statistics related to the performance of the combined sewer system can be computed to develop an understanding of its strengths and weaknesses which, in turn, can be used to identify opportunities for improvement.

The average annual volume of runoff (R) is (Adams and Papa, 2000):

$$R = \theta \, \frac{\phi}{\zeta} \, \mathrm{e}^{-\zeta \cdot S_d} = (104) \frac{(0.50)}{(0.200)} \, \mathrm{e}^{-(0.200)(2.0)} = 174 \text{ mm} \tag{1}$$

The average annual number of CSOs or spills (n_s) is (Adams and Papa, 2000):

$$n_s = \theta \cdot \left[\frac{\frac{\lambda}{\Omega}}{\frac{\lambda}{\Omega} + \frac{\zeta}{\phi}} \right] \cdot \left[\frac{\frac{\psi}{\Omega} + \frac{\zeta}{\phi} \, e^{-\left(\frac{\psi}{\Omega} + \frac{\zeta}{\phi}\right) \cdot S_A}}{\frac{\psi}{\Omega} + \frac{\zeta}{\phi}} \right] e^{-\zeta \cdot S_d} \tag{2}$$

where S_A is the storage capacity of a storage element located immediately upstream of the interceptor sewer which, for this case, is zero and Equation 2 becomes:

$$n_s = \theta \cdot \left[\frac{\frac{\lambda}{\Omega}}{\frac{\lambda}{\Omega} + \frac{\zeta}{\phi}} \right] e^{-\zeta \cdot S_d} = (104) \cdot \left[\frac{\frac{0.282}{0.167}}{\frac{0.282}{0.167} + \frac{0.200}{0.50}} \right] e^{-(0.200)(2.0)} = 56.4 \text{ spills per year}$$

The average annual spill volume (P_u) is (Adams and Papa, 2000):

$$P_u = \frac{\phi}{\zeta} \cdot n_s = \frac{(0.50)}{(0.200)} \cdot (56.4) = 141 \text{ mm} \tag{3}$$

$$P_u = 141 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} \times 400 \text{ ha} \times \frac{10^4 \text{ m}^2}{\text{ha}} = 564'000 \text{ m}^3$$

The average annual fraction of runoff controlled (C_R) is (Adams and Papa, 2000):

$$C_R = 1 - \frac{P_u}{R} = 1 - \frac{141}{174} = 19.0\% \tag{4}$$

The average annual mass of TSS discharged to the Lake during CSOs (M) is:

$$M = \frac{200 \text{ mg}}{\text{L}} \times \frac{1000 \text{ L}}{\text{m}^3} \times 564'000 \text{ m}^3 \times \frac{1 \text{ tonne}}{10^9 \text{ mg}} = 112.8 \text{ tonnes} \tag{5}$$

Runoff detention storage is generally a very effective method of controlling CSOs whereby flows are temporarily detained and released to the interceptor sewer when capacity therein becomes available.

Consider the introduction of a storage element at the downstream end of the combined sewer system with a capacity (S_A) of 2 mm, normalized over the catchment area, being equivalent to 8000 m^3. The average annual number of CSOs (n_s), calculated using Equation 2, becomes:

$$n_s = \theta \cdot \left[\frac{\frac{\lambda}{\Omega}}{\frac{\lambda}{\Omega} + \frac{\zeta}{\phi}} \right] \cdot \left[\frac{\frac{\psi}{\Omega} + \frac{\zeta}{\phi} e^{-\left(\frac{\psi}{\Omega}+\frac{\zeta}{\phi}\right) \cdot S_A}}{\frac{\psi}{\Omega} + \frac{\zeta}{\phi}} \right] e^{-\zeta \cdot S_d}$$

$$= (104) \cdot \left[\frac{\frac{0.282}{0.167}}{\frac{0.282}{0.167} + \frac{0.200}{0.50}} \right] \cdot \left[\frac{\frac{0.0230}{0.167} + \frac{0.200}{0.50} e^{-\left(\frac{0.0230}{0.167}+\frac{0.200}{0.50}\right) \cdot (2.0)}}{\frac{0.0230}{0.167} + \frac{0.200}{0.50}} \right] e^{-(0.200)(2.0)} \qquad (6)$$

$$= 28.7 \text{ spills per year}$$

The remaining performance characteristics are as follows, calculated using Equations 3-5:

- average annual spill volume, P_u = 71.8 mm (287’200 m^3)
- average annual fraction of runoff controlled, C_R = 58.7%
- average annual mass of TSS discharged to the lake during CSOs, M = 57.4 tonnes

It can therefore be seen that a significant performance improvement can be obtained by introducing a relatively small amount of detention storage. Due to the compact and efficient form of the analytical probabilistic models used to calculate system performance, results can be obtained quickly. Moreover, a range of storage values can be analysed with little effort while providing the analyst with a good understanding of the effectiveness of this measure at different levels of implementation. Using Equation 6, a relationship between the average annual number of CSOs and the storage volume can be developed and is illustrated graphically in Figure 2.

As can be seen in the figure, there is a significant reduction (improvement) in the number of CSOs once storage is introduced to the system; however, the relative improvement decreases as storage values increase. This type of relationship is useful in establishing practical or effective design ranges. Based on the figure, it would appear that storage values of 3.0 to 4.0 mm would be the most effective while beyond this range there is limited benefit to be gained.

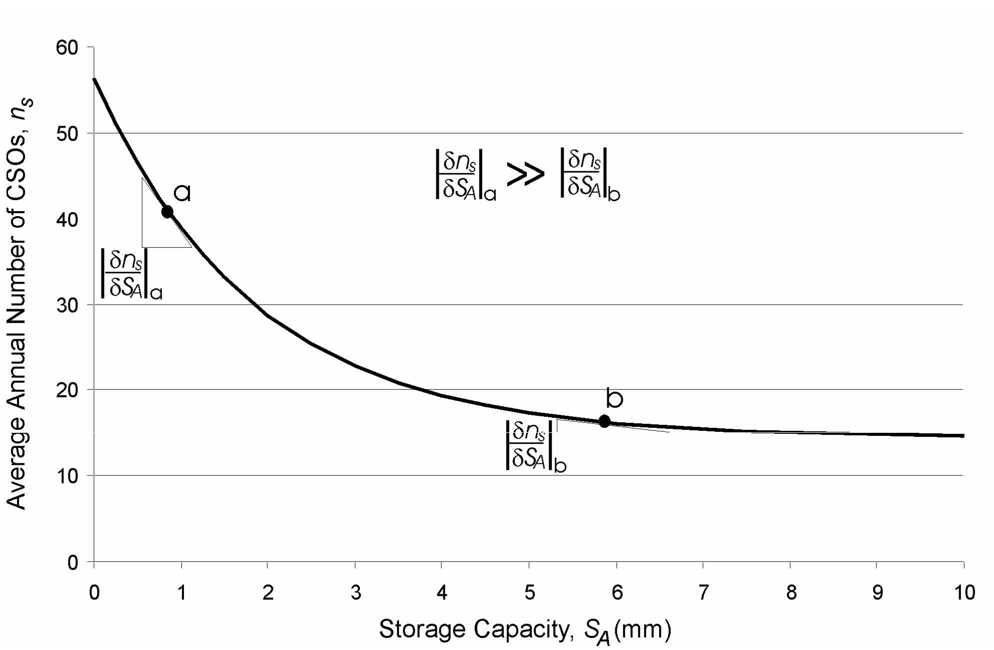

Figure 2. Relationship between average annual number of CSOs and storage volume.

Another technique for improving system performance may be to increase the capacity of the interceptor sewer. For purposes of illustration, consider an increase in interceptor capacity from the current 2 × DWF to 3 × DWF, in addition to the storage element (i.e. S_A = 2.0 mm) introduced earlier. Therefore, the residual capacity in the interceptor sewer to convey storm drainage (Ω) is then 2 × DWF (i.e. 3 × DWF – DWF) or 0.333 mm/h. Using this, the system performance characteristics, calculated using Equations 2-5, become:

- average annual number of CSOs, n_s = 22.8
- average annual spill volume, P_u = 57.0 mm (228'000 m^3)
- average annual fraction of runoff controlled, C_R = 67.2%
- average annual mass of TSS discharged to the lake during CSOs, M = 45.6 tonnes

Again, due to the compact, closed-form model expressions, several relationships between system performance parameters and design variables can be developed, both numerically and graphically. Of particular value in the analysis of system improvement options is the understanding of trade-offs between design (decision) variables, such as storage volume as discussed earlier, and increased interceptor capacity. Expressions which relate these two variables for a given level of service have been developed for various performance parameters. Figure 3 presents an example of such a relationship for runoff control (C_R) and illustrates the combinations of the design variables required to achieve each level of service.

The curves in the figure are referred to as isoquants and represent varying levels of service. The general isoquant expression for this application is (Adams and Papa, 2000):

$$S_A = -\frac{\Omega\,\phi}{\psi\,\phi + \zeta\,\Omega} \cdot \ln\left\{\frac{\phi}{\zeta} \cdot \left[(1 - C_R) \cdot \left(1 + \frac{\zeta\,\Omega}{\lambda\,\phi}\right) \cdot \left(\frac{\psi}{\Omega} + \frac{\zeta}{\phi}\right) - \frac{\psi}{\Omega}\right]\right\} \tag{7}$$

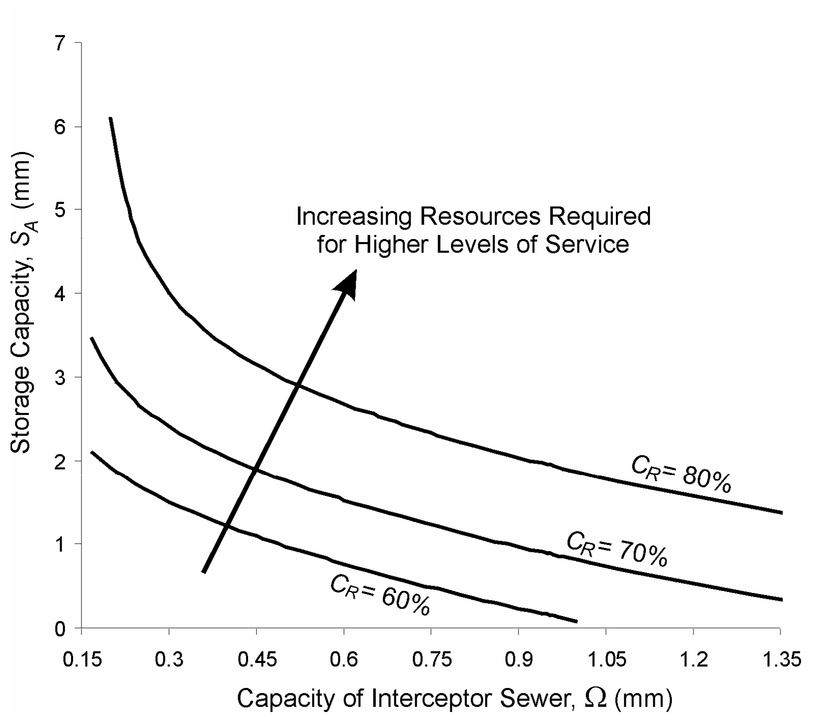

Figure 3. Isoquants of runoff control.

From the figure, it is evident that there is a distinct bend in the C_R = 80% isoquant curve, for example, which defines a region of values where effective combinations of storage and interceptor capacity increases are located. This figure also shows the relative increases in resources required to obtain different levels of service, defined in this case as the average annual fraction of runoff controlled.

The next step in a rational approach to developing an improvement strategy for the combined sewer system under consideration is to incorporate the element of cost to assist in evaluating the value of trade-offs between additional storage capacity and interceptor capacity. The total cost (*TC*) of system improvements can be represented as follows:

$$TC = C_S + C_R \tag{8}$$

where, for illustration purposes, the following expressions can be used to represent the cost of implementing downstream storage (C_S) and interceptor capacity (C_Ω), respectively:

$$C_S = a + b \cdot S_A = \$3'000'000 + \$200'000 \cdot S_A \tag{9}$$

$$C_{\Omega} = c + d \cdot (\Omega' - \Omega) = \$250'000 + \$1'000'000 \cdot (\Omega' - \Omega) \tag{10}$$

where *Ω*' is the total residual capacity of the interceptor sewer. That is, the incremental interceptor sewer capacity is the difference between *Ω*' and *Ω*, the latter term representing the residual capacity of the sewer prior to any improvements.

Substituting Equations 9 and 10 into 8 and rearranging, a relationship can be developed between storage capacity and increased interceptor capacity for any given total cost, referred to as an isocost curve, as follows:

$$S_A = \frac{TC - 3'250'000 - 1'000'000 \cdot (\Omega' - \Omega)}{200'000} \tag{11}$$

Given the cost function, the selection of an appropriate combination of storage capacity and increased interceptor capacity can be approached in one of two ways: (i) the optimal combination can be selected based on a set budgetary amount or (ii) the optimal combination can be set based on a specific performance target. Both these approaches are illustrated graphically in Figure 4.

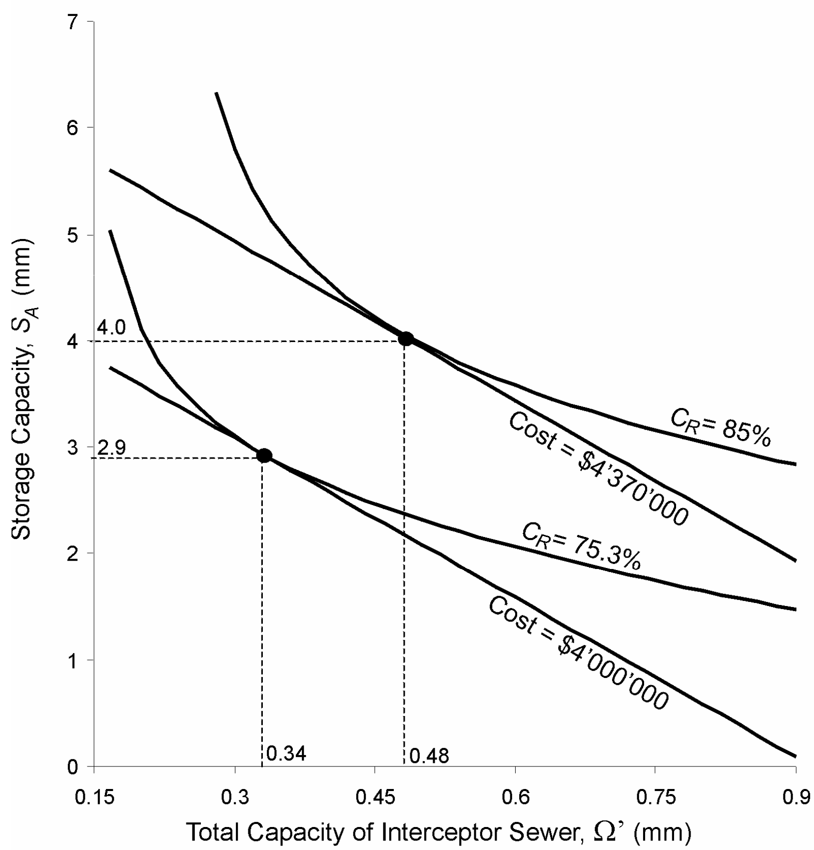

Figure 4. Selection of optimal design variables.

Given a fixed budget of \$4’000’000, the maximum degree of runoff control that can be obtained is estimated to be C_R = 75.3%, based on the selection of 2.9 mm (11’600 m^3) of storage and an increase in interceptor capacity of approximately 0.17 mm/h (i.e. 0.34 mm/h – 0.167 mm/h) or approximately 1 × DWF. Given this design configuration, the system performance characteristics become:

- average annual number of CSOs, n_s = 17.2
- average annual spill volume, P_u = 43.0 mm (172’000 m^3)
- average annual fraction of runoff controlled, C_R = 75.3%
- average annual mass of TSS discharged to the lake during CSOs, M = 34.4 tonnes

In order to achieve the average annual runoff control target of 85%, the optimal values for storage capacity and additional interceptor capacity are 4.0 mm (16’000 m^3) and approximately 0.33 mm/h (i.e. 0.50 mm/h – 0.167 mm/h), respectively. The cost of implementation is calculated to be \$4’370’000. Based on this combination, the system performance characteristics become:

- average annual number of CSOs, n_s = 10.6
- average annual spill volume, P_u = 26.0 mm (106’000 m^3)
- average annual fraction of runoff controlled, C_R = 85%
- average annual mass of TSS discharged to the lake during CSOs, M = 21.2 tonnes

In summary, the following design conditions have been analysed in this example problem, and their respective performance statistics are provided in Table 1 and Figure 5 to illustrate the progression made in developing an appropriate system improvement strategy:

1) original condition or base case;
2) introduction of 2.0 mm of detention storage;
3) addition of 1 × DWF capacity to interceptor sewer;
4) optimization of storage and increased interceptor capacity based on fixed budget;
5) optimization of storage and increased interceptor capacity based on performance target.

Table 1. Summary of performance characteristics.

Design Condition	n_s	P_u (m^3)	C_R (%)	M (tonnes)
1	56.4	564’000	19.0	112.8
2	28.7	287’000	58.7	57.4
3	22.8	228’000	67.2	45.6
4	17.2	172’000	75.3	34.4
5	10.6	106’000	85	21.2

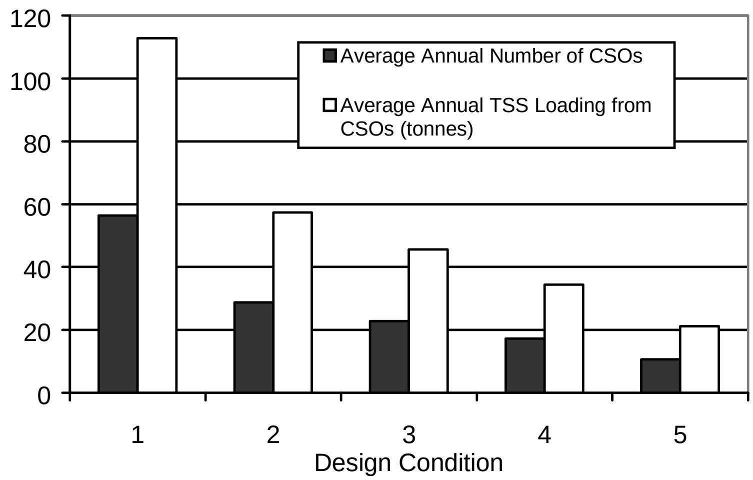

Figure 5. Performance characteristics under alternative design conditions.

CASE STUDY: STORMWATER QUALITY CONTROL POND PERFORMANCE

Stormwater runoff is currently recognized as a significant source of pollutants that degrade receiving waters. In this case study, the use of analytical probabilistic models in conjunction with models for estimating nutrient (i.e. phosphorus and nitrogen) removal is presented to illustrate how the quality control performance of a stormwater management (SWM) pond can be estimated (Valdor Engineering Inc., 2001).

The study area covers 125 ha of commercial/industrial development in the City of Brampton, Ontario, Canada, northwest of Toronto. Much of the area was developed prior to the implementation of regulations on stormwater runoff control and, therefore, runoff from these lands currently discharges directly to Fletchers Creek. The local municipality and conservation authority have made the enhancement and improvement of this watercourse a priority due to a history of water quality and erosion problems.

In 2001, development proposals were submitted to the City to develop the last 13.55 hectares of land in this catchment, consisting of a 9.55 hectare site and a 4.0 hectare site. Initial comments received from the regulatory agencies included the requirement of an on-site SWM facility to service the larger site and for oil-grit separation units to treat the smaller site.

As the requirement for the pond resulted in the loss of prime commercial retail land at a prominent intersection, alternatives were investigated. It was discovered that some City-owned land located in the valley adjacent to Fletchers Creek was otherwise unoccupied and that a diversion sewer could convey flows from both the proposed developments, plus the extensive upstream drainage area which received no treatment prior to discharge, into a stormwater management (SWM) facility at this location. Rather than concentrating on providing treatment to 13.55 hectares, there appeared to be the opportunity to service 125 hectares of land, albeit at lower treatment efficiencies.

To identify a SWM control strategy for implementation, three technically feasible alternatives were developed and evaluated. Table 2 presents the relevant statistics for each of the alternatives. The alternatives selected can be summarized as follows:

1) SWM pond located on 9.55 ha parcel, oil-grit separation units on 4.0 ha parcel;
2) SWM pond on City lands, treating 13.55 ha of proposed commercial developments only;
3) SWM pond on City lands, treating 125.0 ha of commercial/industrial development.

Standard practice in the Province of Ontario at the time of writing is to achieve specific levels of TSS removal from stormwater runoff. Moreover, there are standards and guidelines which are typically referred to in order to estimate the TSS removal and, therefore, modeling of this performance parameter was not warranted. In order to further illustrate the effectiveness of the design alternatives, nutrient removal rates were modelled. Since oil-grit separation units provide limited nutrient removal, only 9.55 hectares of land are serviced by the SWM pond and thereby benefit from nutrient removal under Alternative 1. It is important to note that all ponds considered in this exercise, being consistent with engineering practice in Ontario, are wet ponds. That is, these ponds contain a permanent pool which cannot drain by gravity and which promote pollutant removal from the water column through the processes of sedimentation, biological decay and nutrient uptake.

Table 2. Relevant statistics of stormwater management (SWM) alternatives.

	Alt. 1	Alt. 2	Alt. 3
Drainage Area Serviced by SWM Pond (ha)	9.55	13.55	125.0
Runoff Coefficient (ϕ)	0.73	0.73	0.73
Permanent Pool Volume (m^3)	1000	1400	3800
Permanent Pool Volume (mm)	10.5	10.3	3.0
Mean Permanent Pool Depth (m)	0.3	0.3	0.3

Models for Pollutant Removal

The modelling of the removal of nutrients such as nitrogen and phosphorus is derived from the work of Reckhow (1988), where the basic expression for pollutant removal efficiency (η) is given by:

$$\eta = 1 - \frac{1}{1 + K \cdot T_w} \tag{12}$$

where K is the nutrient trapping parameter (yr^{-1}) and T_w is the hydraulic residence time (years) and is calculated by dividing the volume of the permanent pool by the average annual runoff entering the SWM pond. Moreover, the trapping parameters for nitrogen (K_N) and phoshorus (K_P) are given by the following expressions (Reckhow, 1988):

$$K_N = 0.67 \times T_w^{-0.75} \tag{13}$$

$$K_P = 3.0 \times P_{in}^{0.53} \times T_w^{-0.75} \times z^{0.58} \tag{14}$$

where P_{in} is the influent phosphorus concentration, taken herein to be 0.62 mg/L (Adams and Papa, 2000) and z is the mean depth of the permanent pool (m), values for which are provided in Table 2.

It can therefore be seen from the above expressions that an estimate of the average annual volume of runoff entering the SWM pond is required in order to determine the hydraulic residence time and, further, to determine pollutant removal efficiencies. To this end, analytical probabilistic models were employed using the same meteorological parameters as the previous example (due to the proximity of the Brampton to Toronto). Based on aerial photography of the catchment and its land uses, the hydrological parameters of runoff coefficient and depression storage were taken to be 0.73 and 1.0 mm, respectively.

The average annual nitrogen loading rate is taken to be 13.5 kg/ha/yr (NVPDC, 1979) which corresponds to a loading from the 125 ha catchment to the receiver (Fletchers Creek) of 1687.5 kg/yr. Similarly, the average annual phosphorus loading rate is taken to be 1.8 kg/ha/yr (NVPDC, 1979) which corresponds to a loading rate to the receiver of 225 kg/yr.

Alternative 1

For this alternative, all runoff generated over the 9.55 ha parcel is processed by the SWM pond. The average annual volume of runoff is calculated to be (Equation 1):

$$R = \theta \frac{\phi}{\zeta} \mathrm{e}^{-\zeta \cdot S_d} = (104) \frac{(0.73)}{(0.200)} \mathrm{e}^{-(0.200)\cdot(1.0)} = 311\,\mathrm{mm} \tag{15}$$

As noted in Table 2, the permanent pool volume of the SWM pond is 1000 m^3 being equivalent to 10.5 mm uniformly distributed over the catchment area. The corresponding hydraulic residence time is thus computed to be:

$$T_w = \frac{10.5\,\mathrm{mm}}{311\,\mathrm{mm/yr}} = 0.0338\ \mathrm{years}$$

The nitrogen trapping parameter (K_N) and removal efficiency (η_N) are calculated using Equations 13 and 12, respectively, as follows:

$$K_N = 0.67 \times T_w^{-0.75} = 0.67 \times (0.0338)^{-0.75} = 8.5$$

$$\eta_N = 1 - \frac{1}{1 + K_N \cdot T_w} = 1 - \frac{1}{1 + (8.5)\cdot(0.0338)} = 22.3\%$$

Since the SWM pond in this alternative services only 9.55 ha of the 125 ha catchment, the effective nitrogen removal efficiency across the entire catchment is calculated as:

$$(\eta_N)_{effective} = \frac{(22.3\%)\cdot(9.55)+(0\%)\cdot(125-9.55)}{125} = 1.7\%$$

Similarly, the phosphorus trapping parameter (K_P) and removal efficiency (η_P) are calculated using Equations 14 and 12, respectively, as follows:

$$K_P = 3.0 \times P_{in}^{0.53} \times T_w^{-0.75} \times z^{0.58} = 3.0 \cdot (0.62)^{0.53} \cdot (0.0338)^{-0.75} \cdot (0.3)^{0.58} = 14.7$$

$$\eta_P = 1 - \frac{1}{1 + K_P \cdot T_w} = 1 - \frac{1}{1+(14.7)\cdot(0.0338)} = 33.2\%$$

The effective phosphorus removal efficiency across the entire catchment is calculated as:

$$(\eta_P)_{effective} = \frac{(33.2\%)\cdot(9.55)+(0\%)\cdot(125-9.55)}{125} = 2.5\%$$

Alternative 2

Similarly to Alternative 1, all runoff generated over the 13.55 ha parcel is processed by the SWM pond and the average annual volume of runoff was calculated previously as 311 mm (Equation 15).

As noted in Table 2, the permanent pool volume of the SWM pond is 1400 m^3 being equivalent to 10.3 mm uniformly distributed over the catchment area. The corresponding hydraulic residence time is thus computed to be:

$$T_w = \frac{10.3\,\text{mm}}{311\,\text{mm/yr}} = 0.0331\,\text{years}$$

Following the same procedure as above, the nitrogen and phosphorus removal rates for the pond are calculated to be 22.2% and 33.0%, respectively. Similarly to the previous alternative, the SWM pond in this case services only 13.55 ha of the 125 ha catchment, the effective nitrogen and phosphorus removal efficiencies across the entire catchment are thus calculated to be 2.4% and 3.6%, respectively.

Alternative 3

This alternative involves the construction of a sewer to divert flows from the storm sewer system draining the 125 ha catchment to a SWM pond to be constructed adjacent to Fletchers Creek. The size and capacity of the diversion sewer are 1800 mm in diameter and 6.3 m^3/s, respectively. It is noted that this capacity (Ω) is equivalent to 18.1 mm/h, normalized across the entire 125 ha catchment. Flows in excess of the diversion sewer's capacity are by-passed into the existing downstream section of the storm sewer. A schematic representation of the system is provided in Figure 6.

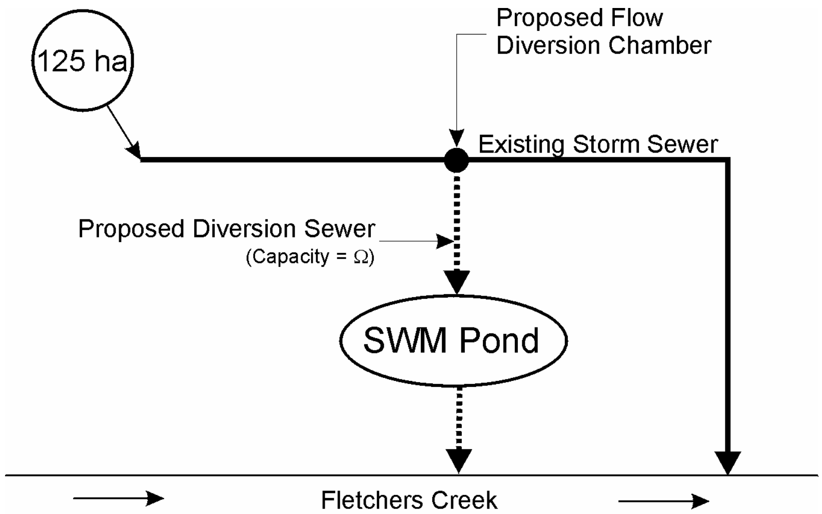

Figure 6. Schematic representation of Alternative 3.

As determined above (Equation 15), the average annual volume of runoff from the catchment is equivalent to 311 mm, normalized across the 125 ha catchment. The amount of runoff that is processed by the SWM pond can be determined by first calculating the average annual fraction of runoff controlled, accounting for the limited capacity of the diversion sewer, as follows (Adams and Papa, 2000):

$$C_R = 1 - \left[\frac{\frac{\lambda}{\Omega}}{\frac{\lambda}{\Omega} + \frac{\zeta}{\phi}} \right] = 1 - \left[\frac{\frac{0.282}{18.1}}{\frac{0.282}{18.1} + \frac{0.200}{0.73}} \right] = 94.6\% \tag{16}$$

Therefore, the average annual volume of runoff processed by the pond is 294 mm (i.e. 311 mm × 94.6%). From Table 2, the permanent pool volume of the pond is 3800 m^3 or 3.0 mm, normalized across the catchment. The corresponding hydraulic residence time is computed to be:

$$T_w = \frac{3.0\,\text{mm}}{294\,\text{mm/yr}} = 0.0102\ \text{years}$$

Following the same procedure as above, the nitrogen and phosphorus removal rates for the pond are calculated to be 17.6% and 26.9%, respectively. It is noted that these figures are representative of the entire catchment and therefore no adjustment is necessary.

Summary of Results

Although an understanding of the nutrient removal characteristics of the SWM alternatives analysed was not required as part of the approvals process, the modeling results were certainly useful in comparing the alternatives. Furthermore, this exercise did not require a significant increase in the level of effort exerted to obtain approvals, due to the ease of computation afforded using the analytical probabilistic models.

The modeling results are summarized in Table 3 and are presented graphically in Figure 7.

Table 3. Summary of modeling results, reported as annual averages.

Alternative	Nitrogen		Phosphorus	
	Mass Removed (kg)	Efficiency (%)	Mass Removed (kg)	Efficiency (%)
1	28.7	1.7	5.6	2.5
2	40.5	2.4	8.1	3.6
3	297	17.6	60.5	26.9

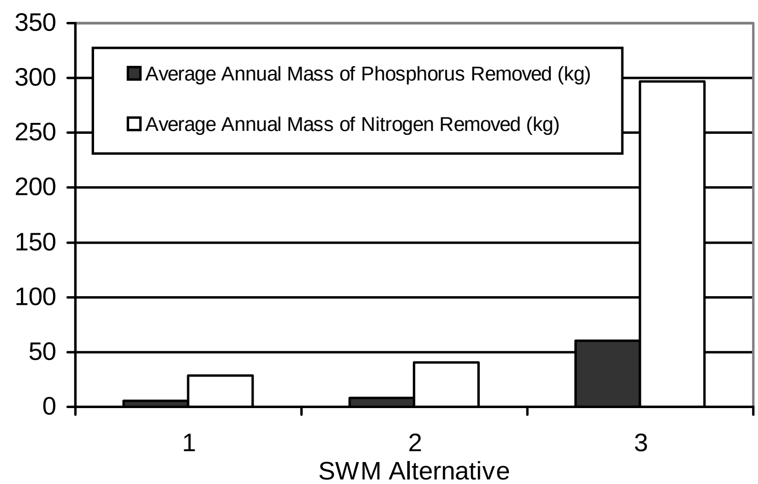

Figure 7. Performance characteristics of alternatives.

From the modeling results it is clear that Alternative 3 provided the most pollution control of those considered, in terms of nitrogen and phosphorus removal. It is noted that similar results were obtained for TSS removal, although not reported herein. More importantly, this solution will place a SWM on City-owned lands located in a valley which would otherwise remain vacant; the City has plans to construct a park trail around the pond along the valley to connect to other trail systems. For the developer, this solution allows for full development of the prime commercial retail lands.

Although Alternative 3 is the most effective in terms of pollution control performance, it is also the most costly. Final approval of the preferred SWM strategy was contingent upon financial arrangements satisfactory to both the City and developer. In the end, approximately 50% of the SWM pond construction costs will be paid for by the developer while the remainder will be derived from the public purse. (The proposed commercial developments occupy less than 11% of the catchment benefiting from treatment in the SWM pond.) It is noted that the developer's financial contribution, however, is more-or-less consistent with development charge policies in place in other jurisdictions and, therefore, this financial commitment was not overly onerous, especially considering the benefit accrued from being able to develop the entire parcel of land.

SOFTWARE

This modeling technology has been incorporated into a formal software package (Walkovich and Adams, 1991). Due to the mathematical simplicity of this modelling approach (i.e. closed-form equations with no iteration or time series computations required), the model expressions can be easily incorporated into customized code or a spreadsheet application to suit the specific needs of the practitioner or project under consideration.

CONCLUSIONS

As an alternative to continuous simulation models, analytical probabilistic models provide a simpler means of assessing the performance of urban drainage systems and their control options for screening or planning level analyses. The name of this modelling approach is derived from:

1) the ability to express the long-term performance of urban drainage systems with computationally efficient, closed-form, mathematical expressions with relatively few input parameters (Analytical);
2) the probabilistic representation of long-term rainfall characteristics derived from the statistical analysis of long-term rainfall records, and the resulting probabilistic representation of urban drainage system performance (Probabilistic).

This paper presents detailed illustrative examples of the usage of this modeling approach to solving typical urban drainage system problems. It is noted that what is presented herein is only a sample of the mathematical expressions and analytical capabilities of these models. Meaningful statistics relating to system performance are shown herein to be easily computed, typically with a hand-held calculator for single-valued results. The power behind this approach, however, is the ability to quickly generate ranges of results for purposes of conducting sensitivity analyses as well as the optimization of design variables through the incorporation of cost functions.

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